Middlebury

CSCI 201: Data Structures

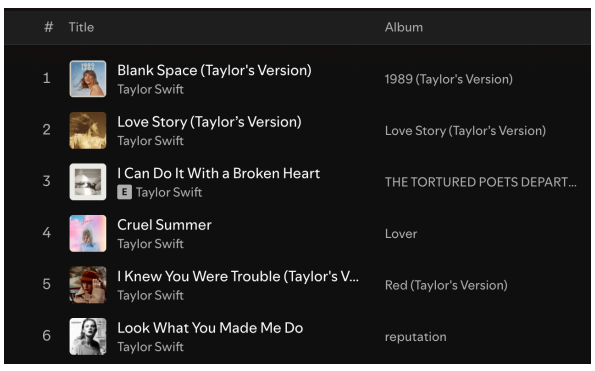
Fall 2025

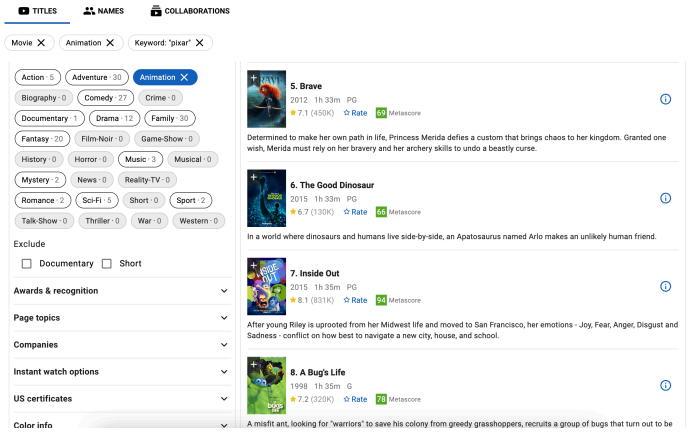
Lecture 4R: Sorting

Goals for today:

- Analyze the runtime of sorting algorithms including **selection sort** and **insertion sort**.
- Work with your groups to implement a sorting algorithm!
- Identify properties of sorting algorithms: **in-place**, **stable**.
- Customize how sorting is done for our own objects.
- Describe the steps in **bucket sort** and **radix sort**.
- Differentiate between the **best**, **worst** and **average** case runtime of an algorithm.

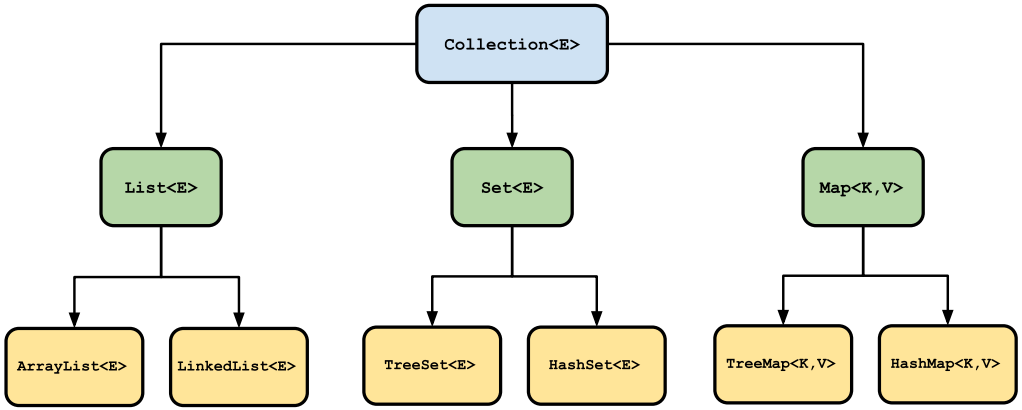
Why is sorting important?





Review worksheet from last class: determine the big-O bound for various methods

The `Collections` framework has built-in methods to `sort`.

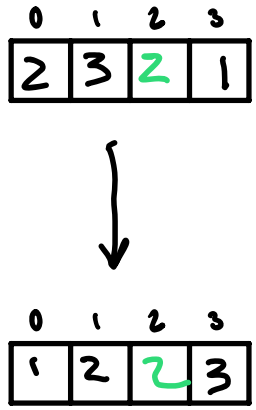


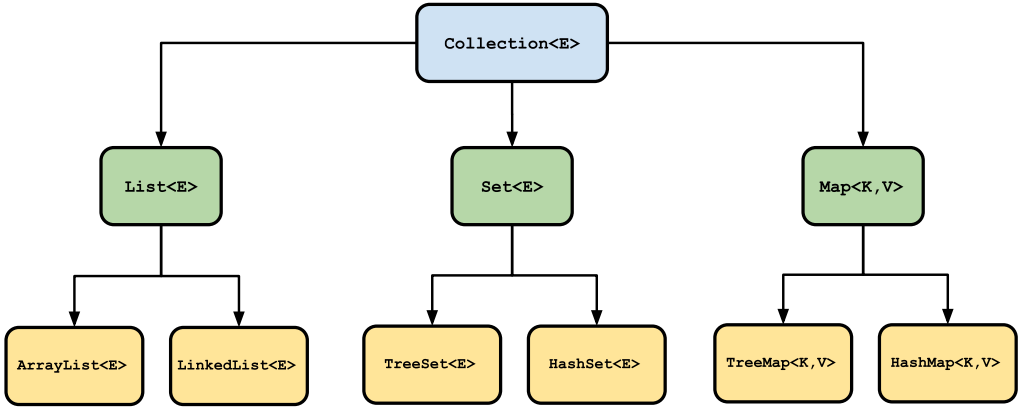

```
public static void sort(List<T> list)
```

Sorts the specified list into ascending order, according to the natural ordering of its elements. All elements in the list must implement the `Comparable` interface.

This sort is guaranteed to be stable: equal elements will not be reordered as a result of the sort.

The `Collections` framework has built-in methods to `sort`.






```
public static void sort(List<T> list)
```

Sorts the specified list into ascending order, according to the natural ordering of its elements. All elements in the list must implement the `Comparable` interface.

This sort is guaranteed to be stable: equal elements will not be reordered as a result of the sort.

The `Arrays` class also has built-in `static` methods to `sort` which can be used for fixed-size arrays.

static void	**sort**(double[] a) Sorts the specified array into ascending numerical order.
static void	**sort**(double[] a, int fromIndex, int toIndex) Sorts the specified range of the array into ascending order.
static void	**sort**(float[] a) Sorts the specified array into ascending numerical order.
static void	**sort**(float[] a, int fromIndex, int toIndex) Sorts the specified range of the array into ascending order.
static void	**sort**(int[] a) Sorts the specified array into ascending numerical order.
static void	**sort**(int[] a, int fromIndex, int toIndex) Sorts the specified range of the array into ascending order.
static void	**sort**(long[] a) Sorts the specified array into ascending numerical order.
static void	**sort**(long[] a, int fromIndex, int toIndex) Sorts the specified range of the array into ascending order.
static void	**sort**(Object[] a) Sorts the specified array of objects into ascending order, according to the **natural ordering** of its elements.
static void	**sort**(Object[] a, int fromIndex, int toIndex) Sorts the specified range of the specified array of objects into ascending order, according to the **natural ordering** of its elements.
static void	**sort**(short[] a) Sorts the specified array into ascending numerical order.
static void	**sort**(short[] a, int fromIndex, int toIndex) Sorts the specified range of the array into ascending order.
static <T> void	**sort**(T[] a, **Comparator**<? super T> c) Sorts the specified array of objects according to the order induced by the specified comparator.
static <T> void	**sort**(T[] a, int fromIndex, int toIndex, **Comparator**<? super T> c) Sorts the specified range of the specified array of objects according to the order induced by the specified comparator.

https://docs.oracle.com/javase/8/docs/api/java/util/Arrays.html

Sorting Algorithm #1 (Selection Sort):

Main idea: Repeatedly *select* the next smallest element and place it in its final position. Maintain sorted elements on the left (of some imaginary divider) and unsorted elements on the right.

1. Find the smallest element in unsorted part.
2. Swap this smallest element with the element to the right of this divider.
3. Move the divider to the right (by one) and go back to Step 1.

Sorting Algorithm #1 (Selection Sort):

Main idea: Repeatedly *select* the next smallest element and place it in its final position. Maintain sorted elements on the left (of some imaginary divider) and unsorted elements on the right.

1. Find the smallest element in unsorted part.
2. Swap this smallest element with the element to the right of this divider.
3. Move the divider to the right (by one) and go back to Step 1.

```
1  public static void sort(int[] items) {
2    for (int i = 0; i < items.length; i++) {
3      int minValue = items[i];
4      int minIndex = i;
5      for (int j = i + 1; j < items.length; j++) {
6        if (items[j] < minValue) {
7          minValue = items[j];
8          minIndex = j;
9        }
10     }
11     items[minIndex] = items[i];
12     items[i] = minValue;
13   }
14 }
```

Sorting Algorithm #1 (Selection Sort):

Main idea: Repeatedly *select* the next smallest element and place it in its final position. Maintain sorted elements on the left (of some imaginary divider) and unsorted elements on the right.

1. Find the smallest element in unsorted part.
2. Swap this smallest element with the element to the right of this divider.
3. Move the divider to the right (by one) and go back to Step 1.

```
1  public static void sort(int[] items) {
2    for (int i = 0; i < items.length; i++) {
3      int minValue = items[i];
4      int minIndex = i;
5      for (int j = i + 1; j < items.length; j++) {
6        if (items[j] < minValue) {
7          minValue = items[j];
8          minIndex = j;
9        }
10     }
11     items[minIndex] = items[i];
12     items[i] = minValue;
13   }
14 }
```

Sorting Algorithm #1 (Selection Sort):

Main idea: Repeatedly *select* the next smallest element and place it in its final position. Maintain sorted elements on the left (of some imaginary divider) and unsorted elements on the right.

1. Find the smallest element in unsorted part.
2. Swap this smallest element with the element to the right of this divider.
3. Move the divider to the right (by one) and go back to Step 1.

```
1  public static void sort(int[] items) {
2    for (int i = 0; i < items.length; i++) {
3      int minValue = items[i];
4      int minIndex = i;
5      for (int j = i + 1; j < items.length; j++) {
6        if (items[j] < minValue) {
7          minValue = items[j];
8          minIndex = j;
9        }
10     }
11     items[minIndex] = items[i];
12     items[i] = minValue;
13   }
14 }
```

Sorting Algorithm #1 (Selection Sort):

Main idea: Repeatedly *select* the next smallest element and place it in its final position. Maintain sorted elements on the left (of some imaginary divider) and unsorted elements on the right.

1. Find the smallest element in unsorted part.
2. Swap this smallest element with the element to the right of this divider.
3. Move the divider to the right (by one) and go back to Step 1.

```
1  public static void sort(int[] items) {
2    for (int i = 0; i < items.length; i++) {
3      int minValue = items[i];
4      int minIndex = i;
5      for (int j = i + 1; j < items.length; j++) {
6        if (items[j] < minValue) {
7          minValue = items[j];
8          minIndex = j;
9        }
10     }
11     items[minIndex] = items[i];
12     items[i] = minValue;
13   }
14 }
```

Sorting Algorithm #1 (Selection Sort):

Main idea: Repeatedly *select* the next smallest element and place it in its final position. Maintain sorted elements on the left (of some imaginary divider) and unsorted elements on the right.

1. Find the smallest element in unsorted part.
2. Swap this smallest element with the element to the right of this divider.
3. Move the divider to the right (by one) and go back to Step 1.

```
1  public static void sort(int[] items) {
2    for (int i = 0; i < items.length; i++) {
3      int minValue = items[i];
4      int minIndex = i;
5      for (int j = i + 1; j < items.length; j++) {
6        if (items[j] < minValue) {
7          minValue = items[j];
8          minIndex = j;
9        }
10     }
11     items[minIndex] = items[i];
12     items[i] = minValue;
13   }
14 }
```

Sorting Algorithm #2 (Insertion Sort):

Main idea: Repeatedly *insert* the next element into those that are already sorted. Maintain sorted elements on the left (of some imaginary divider) and unsorted elements on the right.

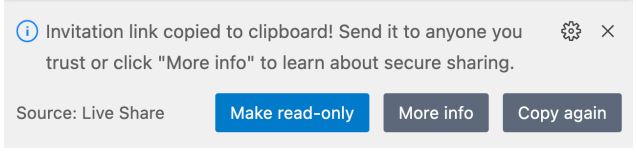
1. Look at first element in unsorted part (to the right of divider).
2. Iteratively swap this into the correct place in the sorted part.
3. Move the divider to the right (by one) and go back to Step 1.

Sorting Algorithm #2 (Insertion Sort):

Main idea: Repeatedly *insert* the next element into those that are already sorted. Maintain sorted elements on the left (of some imaginary divider) and unsorted elements on the right.

1. Look at first element in unsorted part (to the right of divider).
2. Iteratively swap this into the correct place in the sorted part.
3. Move the divider to the right (by one) and go back to Step 1.

Work in groups in `InsertionSort.java`!



Possible implementation of `InsertionSort.java`.

```
public static void sort(int[] items) {
  for (int i = 0; i < items.length; i++) {
    int j = i;
    while (j > 0 && items[j] < items[j - 1]) {
      // swap items at j and j - 1
      int tmp = items[j];
      items[j] = items[j - 1];
      items[j - 1] = tmp;
      j--;
    }
  }
}
```

Runtime analysis of selection and insertion sort.

$1+2+3+\dots+n$

$\frac{n(n+1)}{2}$

```java
// selection sort
public static void sort(int[] items) {
  for (int i = 0; i < items.length; i++) {
    int minValue = items[i];
    int minIndex = i;
    for (int j = i + 1; j < items.length; j++) {
      if (items[j] < minValue) {
        minValue = items[j];
        minIndex = j;
      }
    }
    items[minIndex] = items[i];
    items[i] = minValue;
  }
}
```

# of comparisons

$i=0 \rightarrow n-1$

$i=1 \rightarrow n-2$

$i=2 \rightarrow n-3$

$\vdots$

$i=n-2 \rightarrow 1$

$i=n-1 \rightarrow 0$

$\frac{n(n-1)}{2}$

$O(n^2)$

worst case

best case

```java
// insertion sort
public static void sort(int[] items) {
  for (int i = 0; i < items.length; i++) {
    int j = i;
    while (j > 0 && items[j] < items[j - 1]) {
      // swap items at j and j - 1
      int tmp = items[j];
      items[j] = items[j - 1];
      items[j - 1] = tmp;
      j--;
    }
  }
}
```

$i=0 \rightarrow 0$

$i=1 \rightarrow 1$

$i=2 \rightarrow 2$

$\vdots$

$i=n-2 \rightarrow n-2$

$i=n-1 \rightarrow n-1$

best case $O(n)$

$O(n^2)$

worst case

What if we want to compare our own custom objects? We have two options.

`public interface` **`Comparable<T>`** public int compareTo(T obj) in your class

This interface imposes a total ordering on the objects of each class that implements it. This ordering is referred to as the class's *natural ordering*, and the class's `compareTo` method is referred to as its *natural comparison method*.

`public interface` **`Comparator<T>`** public int compare(T obj1, T obj2) in another class

A comparison function, which imposes a *total ordering* on some collection of objects. Comparators can be passed to a sort method (such as `Collections.sort` or `Arrays.sort`) to allow precise control over the sort order. Comparators can also be used to control the order of certain data structures (such as `sorted sets` or `sorted maps`), or to provide an ordering for collections of objects that don't have a `natural ordering`.

The ordering imposed by a comparator `c` on a set of elements `S` is said to be *consistent with equals* if and only if `c.compare(e1, e2)==0` has the same boolean value as `e1.equals(e2)` for every `e1` and `e2` in `S`.

What if we want to compare our own custom objects? We have two options.

`public interface` **`Comparable<T>`**

This interface imposes a total ordering on the objects of each class that implements it. This ordering is referred to as the class's *natural ordering*, and the class's `compareTo` method is referred to as its *natural comparison method*.

`public interface` **`Comparator<T>`**

A comparison function, which imposes a *total ordering* on some collection of objects. Comparators can be passed to a sort method (such as `Collections.sort` or `Arrays.sort`) to allow precise control over the sort order. Comparators can also be used to control the order of certain data structures (such as `sorted sets` or `sorted maps`), or to provide an ordering for collections of objects that don't have a `natural ordering`.

The ordering imposed by a comparator `c` on a set of elements `S` is said to be *consistent with equals* if and only if `c.compare(e1, e2)==0` has the same boolean value as `e1.equals(e2)` for every `e1` and `e2` in `S`.

Open `MovieSorter.java` for examples.

`implements`ing `compareTo(Movie otherMovie)` within the `Movie` class so it can be `Comparable`.

```
class Movie implements Comparable {
  public String title;
  public int year;
  public double rating;

  public Movie(String title, int year, double rating) {
    this.title = title;
    this.year = year;
    this.rating = rating;
  }

  public int compareTo(Movie otherMovie) {
    if (rating < otherMovie.rating) {
      return -1;
    } else if (rating > otherMovie.rating) {
      return 1;
    }
    return 0;
  }

  public String toString() {
    return title + " (" + year + "), rating = " + rating;
  }
}
```

`implements`ing `compare(Movie movie1, Movie movie2)` outside the `Movie` class to create a `Comparator`.

```
  class MovieYearComparator implements Comparator { // make sure to import java.util.Comparator
    public int compare(Movie movie1, Movie movie2) {
      if (movie1.year < movie2.year) {
        return -1;
      } else if (movie1.year > movie2.year) {
        return 1;
      }
      return 0;
    }
  }


class MovieTitleLengthComparator implements Comparator {
  public int compare(Movie movie1, Movie movie2) {
    if (movie1.title.length() < movie2.title.length()) {
      return -1;
    } else if (movie1.title.length() > movie2.title.length()) {
      return 1;
    }
    return 0;
  }
}

  ...

  // Somewhere else in the code (possibly a PSVM) ...
  // Create a Comparator<T> object and pass it to sort:
  Arrays.sort(movies, new MovieYearComparator()); // sort by year
  Arrays.sort(movies, new MovieTitleLengthComparator()); // sort by title length
```

Sorting Algorithm #3 (Bucket Sort):

Main idea: put items in buckets, sort each bucket, re-assemble.

1. Set up some number of buckets k.
2. **Scatter** all n items into the appropriate bucket.
3. **Sort** each bucket.
4. **Gather** items from buckets into sorted array.

Sorting Algorithm #3 (Bucket Sort):

Main idea: put items in buckets, sort each bucket, re-assemble.

1. Set up some number of buckets k.
2. **Scatter** all n items into the appropriate bucket.
3. **Sort** each bucket.
4. **Gather** items from buckets into sorted array.

Notes:

- Does not require items to be comparable
  (unless using comparison-based sorting for each bucket).
- Works well if the input data is uniformly distributed
  (i.e. buckets evenly sized).
- **Disadvantage:** how to determine number of buckets k?
  (need information about input data).
- Worst-case runtime: $\mathcal{O}(n^2)$.
- Average-case runtime: $\mathcal{O}(n + k)$.
- Not in-place, but stable. In-place algorithms do not need extra space proportional to the input size.

Sorting Algorithm #4 (Radix Sort):

Main idea: similar to bucket sort, use digits to make buckets.

1. Pick a radix (base for each digit; we'll use 10).
2. For each digit d (starting from least significant digit):
    1. Make 10 empty buckets for this digit's possible values (0 - 9).
    2. Get the d^{th} digit of each item
       and put into the appropriate bucket.
    3. Go back through all buckets and put items
       from each bucket back into the original array.

Sorting Algorithm #4 (Radix Sort):

Main idea: similar to bucket sort, use digits to make buckets.

1. Pick a radix (base for each digit; we'll use 10).
2. For each digit d (starting from least significant digit):
    1. Make 10 empty buckets for this digit's possible values (0 - 9).
    2. Get the d^{th} digit of each item
       and put into the appropriate bucket.
    3. Go back through all buckets and put items
       from each bucket back into the original array.

Notes:

- Does not require items to be comparable.
- Worst-case runtime: $\mathcal{O}(n \cdot k)$ (k is the maximum number of digits).
- Average-case runtime: $\mathcal{O}(n \cdot k)$.
- Not in-place, but stable.

See you Friday!

- Keep studying for the programming exam and sign up for your timeslot if you haven't yet!
- Finish Homework 3 by 5PM tonight
- In Lab 4 and Homework 4 we'll practice with implementing some of these sorting algorithms.
- Reminder that Smith (go/smith) has office hours throughout the week and the 201 Course Assistants have drop-in hours in the late afternoons/evenings (go/cshelp).